NEGATIVE SKEWNESS OF PAIRWISE PECULIAR VELOCITY IN THE QUASI-NONLINEAR REGIME: ZEL'DOVICH APPROXIMATION

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ABSTRACT

According to N-body numerical simulations, the pairwise peculiar velocities of galaxies have negative skewness in the quasi-nonlinear regime. To understand its origin, we calculate the probability distribution function of the pairwise velocities using the Zel'dovich approximation, i.e., an approximation for gravitational clustering. The calculated probability distribution function is in good agreement with the result of a N-body simulation. Thus the negative skewness originates in relative motions of galaxies in the clustering process.

Subject headings: cosmology: theory — galaxies: clustering — large-scale structure of universe

1. INTRODUCTION

Let us consider two galaxies separated from each other by the vector $\mathbf{r}$. If their peculiar velocities are $\mathbf{u}_1$ and $\mathbf{u}_2$, the relative velocity $\mathbf{v}$ is defined as

$$\mathbf{v} = \mathbf{u}_2 - \mathbf{u}_1 = v_\parallel \mathbf{n}_\parallel + v_\perp \mathbf{n}_\perp . \tag{1}$$

Here the unit vectors $\mathbf{n}_\parallel$ and $\mathbf{n}_\perp$ are parallel and perpendicular to the separation vector $\mathbf{r}$. We study probability distribution functions (PDFs) of $v_\parallel$ and $v_\perp$, which are defined as pairwise and transverse velocities, respectively ($-\infty < v_\parallel < +\infty$, $0 \leq v_\perp < +\infty$).

The pairwise velocities are of fundamental importance in observational cosmology (Peebles 1993, §20). They serve as a probe of the matter distribution. Their PDF is essential to converting observed data in the redshift space into those in the real space. The recent observation of the 2dF Galaxy Redshift Survey was reported by Peacock et al. (2001).

For the quasi-nonlinear regime of the present-day universe ($r = |\mathbf{r}| \simeq 10^1$–$10^2$ Mpc), N-body numerical simulations have shown that the pairwise velocities have an asymmetric PDF with a negative average and negative skewness (Efstathiou et al. 1988; Fisher et al. 1994; Zurek et al. 1994). This characteristic is attributable to coherent motions of galaxies approaching each other, since gravitational clustering is the predominant process in the quasi-nonlinear regime.

However, with the results of N-body simulations alone, we cannot understand how the asymmetric PDF is related with the physical property of gravitational clustering. To clarify this relation, an analytic treatment is desirable.

The linear perturbation theory, i.e., the most convenient analytic tool, does not allow us to study the above-mentioned asymmetry of the pairwise-velocity PDF. This is because the asymmetry is due to a nonlinear effect (Juszkiewicz, Fisher, & Szapudi 1998). The linear perturbation merely amplifies peculiar velocities of all the galaxies by the same factor. Thus the pairwise-velocity PDF retains its Gaussianity in the initial stage, where motions of the individual galaxies are random and homogeneous.

We employ the Zel'dovich approximation (ZA; Zel'dovich 1970), a Lagrangian quasi-nonlinear perturbation method. In ZA, galaxies cluster together, since each of them moves in the direction of the gravitational force that is determined by the initial density distribution. Thus the velocity and density fields undergo a nonlinear evolution, which is expected to cause the asymmetry of the pairwise-velocity PDF. The simplicity of ZA would help us to discuss analytically the underlying physics of the asymmetric PDF. For the reliability of ZA, see Yoshisato, Matsubara, & Morikawa (1998).

We take special care of the separation range where ZA is applicable. For example, Seto & Yokoyama (1998) used ZA to calculate the pairwise-velocity PDF at separations 1–10 Mpc. Their results did not agree with those of N-body simulations. The above separations are too small for ZA to be valid as demonstrated later.

Throughout this paper, we adopt the Einstein-de Sitter universe with a Hubble constant $H_0 = 50$ km s^{-1} Mpc^{-1}. In §2, an analytic form of the joint PDF of the pairwise and transverse velocities is derived with ZA. In §3, the applicable range of ZA is estimated analytically and numerically. In §4, our ZA calculation is compared with the N-body simulation of Fisher et al. (1994). In §5, the negative skewness is explained analytically. In §6, we conclude with a remark for future research.

2. PAIRWISE VELOCITY DISTRIBUTION

2.1. Analytical Form

Using ZA, we newly derive an analytic form of the joint PDF of the pairwise and transverse velocities at the separation r and the time t, $P(v_\parallel, v_\perp | r, t)$. Here it is implicitly assumed that the vectors $v_\parallel \mathbf{n}_\parallel$ and $v_\perp \mathbf{n}_\perp$ have isotropic distributions. The joint PDF is normalized as

$$\int P\left(v_\parallel, v_\perp | r, t\right) dv_\parallel 2\pi v_\perp dv_\perp = \xi(r,t) + 1, \tag{2}$$

where $\xi(r,t)$ is the two-point correlation function for the number density of galaxies. The pairwise- and transverse-velocity PDFs are obtained by integrating the joint PDF by $v_\perp$ and $v_\parallel$, respectively.

Let us consider a relative motion of two galaxies. Their separation vector $\mathbf{r} = \mathbf{r}(t)$ and relative velocity $\mathbf{v} = \mathbf{v}(t)$ are written with ZA as

$$\begin{aligned} \mathbf{r} &= \mathbf{x}_2 - \mathbf{x}_1 = \mathbf{r}_i + \frac{D}{\dot{D}_i}\mathbf{v}_i, \\ \mathbf{v} &= \mathbf{u}_2 - \mathbf{u}_1 = v_\parallel \mathbf{n}_\parallel + v_\perp \mathbf{n}_\perp = \frac{\dot{D}}{\dot{D}_i}\mathbf{v}_i. \end{aligned} \tag{3}$$

Here $D = D(t) \propto t^{2/3}$ is the linear growth factor of the density fluctuation. We set $D = 1$ for the present-day universe. The suffix i indicates quantities at the initial time $t = t_i$. Since the galaxies move along straight lines, the relative motion lies on the plane determined by the vectors $\mathbf{r}_i$ and $\mathbf{v}_i$.

With an increase of the time from t to $t + dt$, a galaxy pair is defined to move from $(v_\parallel, v_\perp, r)$ to $(v'_\parallel, v'_\perp, r')$. From the conservation of the number of the galaxies, we have

$$P\left(v'_\parallel, v'_\perp | r', t + dt\right) dv'_\parallel 2\pi v'_\perp dv'_\perp 4\pi r'^2 dr' = P\left(v_\parallel, v_\perp | r, t\right) dv_\parallel 2\pi v_\perp dv_\perp 4\pi r^2 dr. \tag{4}$$

From equation (3), the time evolution of $dv_\parallel 2\pi v_\perp dv_\perp 4\pi r^2 dr$ is obtained as

$$dv'_\parallel 2\pi v'_\perp dv'_\perp 4\pi r'^2 dr' = \left(1 + 3\frac{\ddot{D}}{\dot{D}}dt\right) dv_\parallel 2\pi v_\perp dv_\perp 4\pi r^2 dr. \tag{5}$$

The solution is

$$dv_\parallel 2\pi v_\perp dv_\perp 4\pi r^2 dr = \left(\frac{\dot{D}}{\dot{D}_i}\right)^3 dv_{\parallel,i} 2\pi v_{\perp,i} dv_{\perp,i} 4\pi r_i^2 dr_i. \tag{6}$$

Thus the joint PDF at the time t is related to the one at the initial time t_i as

$$P\left(v_\parallel, v_\perp | r, t\right) = \left(\frac{\dot{D}_i}{\dot{D}}\right)^3 P\left(v_{\parallel,i}, v_{\perp,i} | r_i, t_i\right). \tag{7}$$

From equation (3), the initial separation r_i is obtained as

$$r_i = \sqrt{\left(r - \frac{D}{\dot{D}} v_\parallel\right)^2 + \left(\frac{D}{\dot{D}} v_\perp\right)^2}. \tag{8}$$

Likewise, the initial relative velocities $v_{\parallel,i}$ and $v_{\perp,i}$ are

$$\begin{aligned} v_{\parallel,i} &= \frac{\dot{D}_i}{\dot{D}} \left(\frac{r v_\parallel}{r_i} - \frac{D}{\dot{D}} \frac{v_\parallel^2 + v_\perp^2}{r_i}\right), \\ v_{\perp,i} &= \frac{\dot{D}_i}{\dot{D}} \frac{r v_\perp}{r_i}, \end{aligned} \tag{9}$$

(see also Appendix A). Seto & Yokoyama (1998) used ZA to derive directly an analytic form of the pairwise-velocity PDF, $P(v_\parallel | r, t)$. We prefer the joint PDF, $P(v_\parallel, v_\perp | r, t)$, which has a more simplified form and provides us with clear information about, e.g., the origin of the negative skewness of the pairwise velocities (see §5).

2.2. Initial Condition

Here we determine the initial condition for $P(v_\parallel, v_\perp | r, t)$. The initial density fluctuation is assumed to be random Gaussian. This is the most simple and standard model (Peebles 1993, §17). The initial peculiar-velocity field is assumed to be homogeneous, isotropic, random, and vorticity-free. The last assumption originates in the fact that, even if vorticity exists in the initial peculiar-velocity field, it decays rapidly during the expansion of the universe (Peebles 1993, §5).

Since the initial density fluctuation is random Gaussian, the corresponding PDF of the pairwise and transverse velocities is also Gaussian (Seto & Yokoyama 1998; see also Appendix B),

$$P\left(v_{\parallel,i}, v_{\perp,i} | r_i, t_i\right) = \frac{1}{\sqrt{(2\pi)^3 \sigma_\parallel^2(r_i) \sigma_\perp^4(r_i)}} \exp\left[-\frac{1}{2}\left(\frac{v_{\parallel,i}^2}{\sigma_\parallel^2(r_i)} + \frac{v_{\perp,i}^2}{\sigma_\perp^2(r_i)}\right)\right]. \tag{10}$$

Thus we only have to obtain the initial dispersions of the relative velocities $\sigma_\parallel^2(r_i)$ and $\sigma_\perp^2(r_i)$.

If the peculiar-velocity field is homogeneous, the relative-velocity dispersion is derived from the one-point velocity dispersion and the two-point velocity correlation:

$$\begin{aligned}\sigma_\parallel^2(r_i) &= \langle (u_{\parallel,i}(\mathbf{x}_i+\mathbf{r}_i) - u_{\parallel,i}(\mathbf{x}_i))^2\rangle = 2\langle u_{\parallel,i}^2(\mathbf{x}_i)\rangle - 2\langle u_{\parallel,i}(\mathbf{x}_i)u_{\parallel,i}(\mathbf{x}_i+\mathbf{r}_i)\rangle \\ \sigma_\perp^2(r_i) &= \langle (u_{\perp,i}(\mathbf{x}_i+\mathbf{r}_i) - u_{\perp,i}(\mathbf{x}_i))^2\rangle = 2\langle u_{\perp,i}^2(\mathbf{x}_i)\rangle - 2\langle u_{\perp,i}(\mathbf{x}_i)u_{\perp,i}(\mathbf{x}_i+\mathbf{r}_i)\rangle.\end{aligned} \tag{11}$$

Here $\langle\cdot\rangle$ denotes the ensemble average, and $u_{\parallel,i}$ and $u_{\perp,i}$ are respectively the parallel and perpendicular components of the peculiar velocity $\mathbf{u}_i$. These velocity correlations are related to the power spectrum of the density fluctuation as follows (Górski 1988). The initial peculiar-velocity field is homogeneous, isotropic, random, and vorticity-free. Using a theory for such a vector field (Monin & Yaglom 1975, §12), we have

$$\begin{aligned}\langle u_{\parallel,i}(\mathbf{x}_i)u_{\parallel,i}(\mathbf{x}_i+\mathbf{r}_i)\rangle &= 2\int_0^\infty \left[\frac{\sin(k_ir_i)}{k_ir_i} + 2\frac{\cos(k_ir_i)}{(k_ir_i)^2} - 2\frac{\sin(k_ir_i)}{(k_ir_i)^3}\right]\mathcal{E}_i(k_i)dk_i \\ &= 2\int_0^\infty \left[j_0(k_ir_i) - 2\frac{j_1(k_ir_i)}{k_ir_i}\right]\mathcal{E}_i(k_i)dk_i, \\ \langle u_{\perp,i}(\mathbf{x}_i)u_{\perp,i}(\mathbf{x}_i+\mathbf{r}_i)\rangle &= 2\int_0^\infty \left[-\frac{\cos(k_ir_i)}{(k_ir_i)^2} + \frac{\sin(k_ir_i)}{(k_ir_i)^3}\right]\mathcal{E}_i(k_i)dk_i \\ &= 2\int_0^\infty \left[\frac{j_1(k_ir_i)}{k_ir_i}\right]\mathcal{E}_i(k_i)dk_i.\end{aligned} \tag{12}$$

Here j_0 and j_1 are first-kind spherical Bessel functions,

$$\begin{aligned} j_0(x) &= \frac{\sin x}{x}, \\ j_1(x) &= \frac{\sin x}{x^2} - \frac{\cos x}{x},\end{aligned} \tag{13}$$

and $\mathcal{E}_i(k_i)$ is the power spectrum of the initial peculiar-velocity field,

$$\int \mathcal{E}_i(k_i)dk_i = \frac{1}{2}\left\langle |\mathbf{u}_i(\mathbf{x}_i)|^2\right\rangle. \tag{14}$$

Using the linear perturbation theory, we relate the power spectrum of the initial peculiar-velocity field $\mathcal{E}_i(k_i)$ to that of the initial density fluctuation $\mathcal{P}_i(k_i)$. The linear peculiar-velocity field is described by the linear density field as

$$\mathbf{u}_i(\mathbf{x}_i) = i\frac{\dot{D}_i}{D_i}\int \frac{\mathbf{k}_i}{k_i^2}\tilde{\delta}_i(\mathbf{k}_i)\exp(i\mathbf{k}_i\mathbf{x}_i)\frac{d\mathbf{k}_i}{(2\pi)^{3/2}}, \tag{15}$$

where $\tilde{\delta}_i(\mathbf{k}_i)$ is the Fourier transform of the density contrast $\delta_i(\mathbf{x}_i)$ (Peebles 1993, §21):

$$\delta_i(\mathbf{x}_i) = \int \tilde{\delta}_i(\mathbf{k}_i)\exp(i\mathbf{k}_i\mathbf{x}_i)\frac{d\mathbf{k}_i}{(2\pi)^{3/2}}. \tag{16}$$

The power spectrum $\mathcal{P}_i(k_i)$ of the initial density fluctuation is

$$\left\langle \tilde{\delta}_i(\mathbf{k}_i)\tilde{\delta}_i(\mathbf{k}'_i)^* \right\rangle = \mathcal{P}_i(k_i)\,(2\pi)^3\,\delta(\mathbf{k}_i - \mathbf{k}'_i), \tag{17}$$

where $*$ denotes the complex conjugate. From equations (14), (15), and (17), the power spectrum of the initial peculiar-velocity field is obtained as

$$\mathcal{E}_i(k_i) = 2\pi \left(\frac{\dot{D}_i}{D_i}\right)^2 \mathcal{P}_i(k_i). \tag{18}$$

Therefore, the pairwise- and transverse-velocity dispersions in the initial stage are

$$\sigma_\parallel^2(r_i) = \frac{8\pi}{3}\left(\frac{\dot{D}_i}{D_i}\right)^2 \int \left[1 - 3j_0(k_i r_i) + 6\frac{j_1(k_i r_i)}{k_i r_i}\right] \mathcal{P}_i(k_i) dk_i = \left(\frac{\dot{D}_i}{D_i}\right)^2 R_\parallel^2(r_i), \tag{19}$$

$$\sigma_\perp^2(r_i) = \frac{8\pi}{3}\left(\frac{\dot{D}_i}{D_i}\right)^2 \int \left[1 - 3\frac{j_1(k_i r_i)}{k_i r_i}\right] \mathcal{P}_i(k_i) dk_i = \left(\frac{\dot{D}_i}{D_i}\right)^2 R_\perp^2(r_i). \tag{20}$$

The initial condition for $P(v_\parallel, v_\perp | r, t)$ is given by equations (10), (19) and (20).

Since the peculiar velocity depends on the gravitational potential produced by the density fluctuation, the above equations include the information that galaxies are about to cluster together. In fact, the integration $\int \mathcal{P}_i(k_i) dk_i$ is proportional to the mean gravitational potential due to the initial density fluctuation (Peebles 1993, §21). The quantities $R_\parallel(r_i)$ and $R_\perp(r_i)$ represent characteristic lengths for gradients of the gravitational potential along the separation r_i.

2.3. Power Spectrum of Initial Density Fluctuation

The power spectrum of the initial density fluctuation $\mathcal{P}_i(k_i)$ is adopted from the cold dark matter model, a standard model for structure formation (Peebles 1993, §25):

$$\mathcal{P}_i(k_i) = \frac{B D_i^2 k_i}{\left\{1 + \left[\alpha k_i + (\beta k_i)^{3/2} + (\gamma k_i)^2\right]^\nu\right\}^{2/\nu}}, \tag{21}$$

where $\alpha = 25.6$ Mpc, $\beta = 12$ Mpc, $\gamma = 6.8$ Mpc, and $\nu = 1.13$ for $\Omega_0 = 1$ and $H_0 = 50$ km s^{-1} Mpc^{-1} (Bond & Efstathiou 1984; Efstathiou, Bond, & White 1992). The normalization factor B is determined from the temperature fluctuation of the cosmic microwave background:

$$B = \frac{12\Omega_0^{-1.54} c^4}{5\pi H_0^4} \left(\frac{\mathcal{Q}_{rms}}{T_0}\right)^2. \tag{22}$$

Here the quadrupole fluctuation amplitude $\mathcal{Q}_{rms}$ is set to be $9.5\,\mu$K with the present-day temperature of $T_0 = 2.73$ K. The same power spectrum was adopted as the initial condition in the N-body simulation of Fisher et al. (1994), which is to be compared with our calculation, and in the ZA calculation of Seto & Yokoyama (1998).

Note that there are relations $\mathcal{P}_i(k_i) \propto D_i^2$, $\sigma_\parallel(r_i) \propto \dot{D}_i$, $\sigma_\perp(r_i) \propto \dot{D}_i$, $v_{\parallel,i} \propto \dot{D}_i$, and $v_{\perp,i} \propto \dot{D}_i$ (eqs. [9] and [19]–[21]). Thus the initial condition for $P\left(v_\parallel, v_\perp | r, t\right)$ does not depend on the values of D_i and $\dot{D}_i$.

3. APPLICABLE RANGE OF ZA

Since ZA is merely an approximation, ZA is applicable to a limited range of the separation. In ZA, galaxies continue to move straight, and pass each other even if they gather once. In reality, the gathered galaxies are bounded gravitationally. ZA fails after this so-called shell crossing. Here we estimate the applicable range of ZA. It is determined by the separation below which the influence of the shell crossing is statistically significant. In §3.1, we analytically estimate the application limit. In §3.2, we make a numerical estimation on the basis of the time evolution of the two-point correlation function.

3.1. Analytical Estimation

Let us consider a pair of galaxies which are approaching each other. If its initial separation is r_i, its typical initial pairwise velocity is estimated as $v_{\parallel,i} \simeq -\sigma_\parallel(r_i) = -\dot{D}_i D_i^{-1} R_\parallel(r_i)$. From the definition of ZA (eq. [3]), we have

$$r_i - r \simeq -\frac{D}{\dot{D}_i} v_{\parallel,i} \simeq \frac{D}{D_i} R_\parallel(r_i). \tag{23}$$

The shell crossing occurs if $r = 0$. Thus ZA is valid as far as $r_i \gg D D_i^{-1} R_\parallel(r_i)$. This condition implies $r \simeq r_i$. We replace r_i with r and obtain

$$r \gg \frac{D}{D_i} R_\parallel(r). \tag{24}$$

This is the applicable range of ZA. Figure 1 compares the right side of equation (24) with the left side as a function of the scale r at $D = 1$. It is evident that ZA fails at $r \lesssim 10$ Mpc.

3.2. Numerical Estimation

The shell crossing affects the time evolution of the two-point correlation function. This is because, at a given separation, galaxies become less clustered after the onset of the shell crossing. Using ZA, we numerically calculate the two-point correlation function $\xi(r,t)$ and its time derivative $\partial\xi(r,t)/\partial t$ at $D = 1$ (see also Appendix A). The results are plotted against the separation r in Figure 2. The time growth $\partial\xi(r,t)/\partial t$ of the two-point correlation function is maximal at $r \simeq 20\,\mathrm{Mpc}$ and is zero at $r \simeq 10\,\mathrm{Mpc}$. The latter separation coincides with the separation for $r = DD_i^{-1}R_\parallel(r)$ (§3.1). Thus the ZA's applicable range is $r \gtrsim 20$ Mpc at the time for $D = 1$.

Seto & Yokoyama (1998) studied the pairwise-velocity PDF with ZA at separations r = 1–10 Mpc, which are too small for ZA to be valid. This is the reason why they underestimated the average of the pairwise velocity $\langle v_\parallel \rangle$ by an order of magnitude, compared with the N-body simulation done by Zurek et al. (1994) for the same condition. After the shell crossing, the pairwise-velocity PDF approaches a Gaussian function with a zero average, as recognized by Seto & Yokoyama (1998).

4. COMPARISON WITH N-BODY SIMULATION

With the same initial condition as ours (§2.3), Fisher et al. (1994) conducted a N-body simulation. The pairwise-velocity PDF was presented for $\xi(r,t) = 0.1$, which corresponds to $r \simeq 35$ Mpc at $D = 1$ (Fig. 2). This separation is within the ZA's applicable range (§3). Figure 3 compares the PDF for $\xi(r,t) = 0.1$ in the N-body simulation with the PDF at $r = 35$ Mpc in our ZA calculation. The latter well reproduces the former. Thus the negative average as well as negative skewness of the pairwise velocities observed in N-body simulations is due to gravitational clustering of galaxies. The same conclusion was obtained by Juszkiewicz et al. (1998) using the second-order Eulerian perturbation theory.

The average $\langle v_\parallel \rangle$, standard deviation $\langle (v_\parallel - \langle v_\parallel \rangle)^2 \rangle^{1/2}$, and skewness $\langle (v_\parallel - \langle v_\parallel \rangle)^3 \rangle / \langle (v_\parallel - \langle v_\parallel \rangle)^2 \rangle^{3/2}$ of the pairwise velocities are -190 km s^{-1}, 430 km s^{-1}, and -0.69, respectively, in the N-body simulation. They are -120 km s^{-1}, 480 km s^{-1}, and -0.32, respectively, in our ZA calculation. The average and standard deviation are in satisfactory agreement. However, the skewness in the N-body simulation is more enhanced than that in our ZA calculation. The pairwise-velocity PDF in the N-body simulation has a more pronounced tail in the negative side. This is probably because gravitational acceleration of galaxies in the N-body simulation is more significant than that assumed in ZA, which simply extrapolates the initial velocity $\mathbf{v}_i$ (eq. [3]; see also Juszkiewicz et al. 1998).

5. EXPLANATION FOR NEGATIVE SKEWNESS

The origin of the negative average and skewness is explained analytically with ZA. Since ZA is applicable at $r \gg DD_i^{-1}R_\parallel(r)$, the behavior of $P(v_\parallel, v_\perp|r,t)$ is studied separately for the velocity ranges $|v_\parallel| \ll \dot{D}D_i^{-1}R_\parallel(r)$, $\dot{D}D_i^{-1}R_\parallel(r) \ll |v_\parallel| \ll \dot{D}D^{-1}r$, and $\dot{D}D^{-1}r \ll |v_\parallel|$. The values of $\dot{D}D_i^{-1}R_\parallel(r)$ and $\dot{D}D^{-1}r$ are 530 and 1750 km s^{-1}, respectively, at $r = 35$ Mpc and $D = 1$.

First, we study the velocity range $\dot{D}D_i^{-1}R_\parallel(r) \ll |v_\parallel| \ll \dot{D}D^{-1}r$. Since most of the galaxies have relatively small transverse velocities $v_\perp \simeq \dot{D}\dot{D}_i^{-1}\sigma_\perp(r) \simeq \dot{D}D_i^{-1}R_\perp(r) \simeq \dot{D}D_i^{-1}R_\parallel(r)$, equations (8) and (9) are simplified as

$$r_i \simeq r\left(1 - \frac{D}{\dot{D}}\frac{v_\parallel}{r}\right), \tag{25}$$

$$v_{\parallel,i} \simeq \frac{\dot{D}_i}{\dot{D}}v_\parallel, \tag{26}$$

$$v_{\perp,i} \simeq \frac{\dot{D}_i}{\dot{D}}v_\perp. \tag{27}$$

From these equations, we have

$$r_i(v_\parallel = +V) < r_i(v_\parallel = -V), \tag{28}$$

$$|v_{\parallel,i}(v_\parallel = +V)| = |v_{\parallel,i}(v_\parallel = -V)|, \tag{29}$$

where $V = |v_\parallel|$. Figure 4a shows that the standard deviation $\sigma_\parallel(r_i)$ of the Gaussian function $P(v_{\parallel,i}, v_{\perp,i}|r_i,t_i)$ is larger at a larger separation r_i if $r_i \lesssim 100$ Mpc. Generally, as explained in Figure 4b, a Gaussian function with a larger standard deviation yields a larger value for an argument which exceeds the standard deviation. Hence we expect

$$P\left(v_{\parallel,i}, v_{\perp,i}|r_i', t_i\right) < P\left(v_{\parallel,i}, v_{\perp,i}|r_i'', t_i\right) \quad \text{for} \quad r_i' < r_i'', \tag{30}$$

which leads to

$$P\left(v_\parallel = +V, v_\perp|r,t\right) < P\left(v_\parallel = -V, v_\perp|r,t\right). \tag{31}$$

Thus the PDF is asymmetric in this velocity range.

Second, we study the velocity range $|v_\parallel| \ll \dot{D}D_i^{-1}R_\parallel(r)$. The absolute value of $v_\parallel$ tends to be much smaller than $v_\perp$. Thus the sign of $v_\parallel$ is not so important to the values of r_i and $v_{\parallel,i}$ (eqs. [8] and [9]). It is natural to expect $P(v_\parallel = +V, v_\perp|r,t) \simeq P(v_\parallel = -V, v_\perp|r,t)$. Since the initial PDF is Gaussian, there is a large fraction of galaxies in this velocity range, which corresponds to $|v_{\parallel,i}| \ll \sigma_\parallel(r_i)$.

Third, we study the velocity range $|v_\parallel| \gg \dot{D}D^{-1}r$. Equations (8) and (9) yield $r_i \simeq D\dot{D}^{-1}|v_\parallel|$ and hence $v_{\parallel,i} \simeq -\dot{D}_i\dot{D}^{-1}|v_\parallel|$, which in turn yield $P(v_\parallel = +V, v_\perp|r,t) \simeq P(v_\parallel = -V, v_\perp|r,t)$. The value of $P(v_\parallel = +V, v_\perp|r,t)$ is overestimated in ZA. The velocity range $v_\parallel \gg \dot{D}D^{-1}r$ contains many galaxies that have experienced the shell crossing.

Therefore, the inequality $P(v_\parallel = +V, v_\perp|r,t) < P(v_\parallel = -V, v_\perp|r,t)$ exists in the velocity range

$$\frac{\dot{D}}{D_i}R_\parallel(r) \lesssim |v_\parallel| \lesssim \frac{\dot{D}}{D}r. \tag{32}$$

This asymmetry is responsible for the negative average as well as the negative skewness observed in the ZA's pairwise-velocity PDF, $P(v_\parallel|r,t)$. The upper limit in equation (32) is due to the shell crossing. The pairwise-velocity PDF in the real universe would be asymmetric beyond this limit. On the other hand, the lower limit reflects the fact that the pairwise velocities among galaxies with small peculiar velocities, which have not moved significant distances from their initial positions, retain the initial Gaussian character.

Here we underline that the existence of the asymmetry region (eq.[32]) is due to the condition for applicability of ZA, $r \gg DD_i^{-1}R_\parallel(r)$ (eq. [24]). Thus, whenever ZA is applicable, the resultant pairwise velocities have a negative average and negative skewness.

The dependence of the standard deviation $\sigma_\parallel(r_i)$ on the separation r_i is an assured behavior. It is insensitive to the model for the power spectrum of the initial density fluctuation, as far as the model is a realistic one. The separation dependence stems from the function $1-3j_0(k_ir_i)+6j_1(k_ir_i)/(k_ir_i)$ in the integration of equation (19), which contains information how galaxies cluster together according to the initial gravitational field.

6. CONCLUSION

The pairwise velocities of galaxies are known to exhibit a negative average as well as negative skewness. To understand their origin, we have used ZA to study analytically the pairwise-velocity PDF.

We have estimated that the applicable range of ZA, where we can ignore the influence of shell crossing, is $r \gtrsim 20$ Mpc for the Einstein-de Sitter universe with $H_0 = 50$ km s^{-1} Mpc^{-1} at $D = 1$ (§3).

We have compared our ZA calculation with the result of the N-body simulation done under the same initial condition by Fisher et al. (1994). ZA reproduces successfully the negative average and skewness of the pairwise-velocity PDF. They are surely due to clustering of galaxies (§4).

We have analytically discussed the mechanism to generate the negative average and skewness. They are explained by the dependence of the initial velocity dispersion on the separation, which includes information of galaxy clustering according to the gradient of the initial gravitational potential (§5).

The pairwise-velocity PDF obtained with N-body simulations has a pronounced tail in the negative side (Efstathiou et al. 1988; Fisher et al. 1994; Zurek et al. 1994). However, ZA cannot reproduce this property (§4). It is attributable to gravitational acceleration of galaxies which is not fully incorporated in ZA. To study the origin of the pronounced tail, we would require the higher-order approximations, e.g., post-ZA and post-post-ZA, which use not only the initial velocity $\mathbf{v}_i$ but also the initial acceleration $\dot{\mathbf{v}}_i$ and so on (Bouchet et al. 1995; see also Yoshisato et al. 1998 and references therein). The approximations Padé-post-ZA and Padé-post-post-ZA developed by Matsubara, Yoshisato, & Morikawa (1998) would be also useful.

This paper is in part the result of research of A. Y. toward fulfillment of the requirements of the Ph. D. degree at Ochanomizu University.

A. TIME EVOLUTION OF PDF

To understand the behavior of $P(v_\parallel, v_\perp | r, t)$ in more detail, we derive its time-evolution equation. Let us expand $P(v'_\parallel, v'_\perp | r', t+dt)$ around t using ZA (eqs. [3] and [5]):

$$\begin{aligned} &P\left(v'_\parallel, v'_\perp | r', t+dt\right) \delta v'_\parallel 2\pi v'_\perp \delta v'_\perp 4\pi r'^2 \delta r' \\ &\quad = \left[P\left(v_\parallel, v_\perp | r, t\right) + \frac{\partial P}{\partial v_\parallel}\left(\frac{\ddot{D}}{\dot{D}} v_\parallel + \frac{v_\perp^2}{r}\right) dt + \frac{\partial P}{\partial v_\perp}\left(\frac{\ddot{D}}{\dot{D}} v_\perp - \frac{v_\perp v_\parallel}{r}\right) dt \right. \\ &\qquad \left. + \frac{\partial P}{\partial r} v_\parallel dt + \frac{\partial P}{\partial t} dt \right] \left(1 + 3\frac{\ddot{D}}{\dot{D}} dt\right) \delta v_\parallel 2\pi v_\perp \delta v_\perp 4\pi r^2 \delta r. \end{aligned} \tag{A1}$$

From a comparison between equations (4) and (A1), we obtain the time-evolution equation of the joint PDF as

$$\frac{\partial P}{\partial t} = -\frac{\ddot{D}}{\dot{D}}\left(v_\parallel \frac{\partial P}{\partial v_\parallel} + v_\perp \frac{\partial P}{\partial v_\perp} + 3P\right) - \frac{v_\perp}{r}\left(v_\perp \frac{\partial P}{\partial v_\parallel} - v_\parallel \frac{\partial P}{\partial v_\perp}\right) - v_\parallel \frac{\partial P}{\partial r}. \tag{A2}$$

This equation is equivalent to a collision-less Boltzman equation because ZA ignores interactions among the individual galaxies. The first, second, and third terms represent the gravitational acceleration, rotation, and translation of the galaxies, respectively. However, $(v_\parallel, v_\perp, r)$ do not constitute the generalized coordinates. This fact yields the third term, $3P$, in the parenthesis of the first term (see eq. [5]).

The equation (A2) allows us to discuss time evolutions of various quantities. For example, the integration by $dv_\parallel 2\pi v_\perp dv_\perp$ yields the evolution of $\xi(r,t)$,

$$\frac{\partial}{\partial t}\left(1+\xi(r,t)\right) = \frac{\partial \xi(r,t)}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(1+\xi(r,t)\right)\left\langle v_\parallel\right\rangle\right], \tag{A3}$$

which corresponds to a continuity equation. This is a reasonable result because we have assumed the conservation of the galaxy number (eq. [4]).

B. MEAN PAIRWISE VELOCITY: LINEAR THEORY

With the linear perturbation theory, we obtain the mean pairwise velocity $\langle v_{\parallel,i}\rangle$ in the initial stage (see also Fisher 1995). Equations (15) and (16) yield

$$\langle \delta_i(\mathbf{x}_i)\mathbf{u}_i(\mathbf{x}_i+\mathbf{r}_i)\rangle = i\frac{\dot{D}_i}{D_i}\int \frac{\mathbf{k}_i}{k_i^2}\mathcal{P}_i(k_i)\exp(i\mathbf{k}_i\mathbf{r}_i)d\mathbf{k}_i. \tag{B1}$$

The pairwise component $\langle \delta_i(\mathbf{x}_i)u_{\parallel,i}(\mathbf{x}_i+\mathbf{r}_i)\rangle$ of the above vector is

$$\langle \delta_i(\mathbf{x}_i)u_{\parallel,i}(\mathbf{x}_i+\mathbf{r}_i)\rangle = -4\pi\frac{\dot{D}_i}{D_i}\int k_i j_1(k_i r_i)\mathcal{P}_i(k_i)dk_i. \tag{B2}$$

Using the above relation, the mean pairwise velocity is obtained as a galaxy-number-weighted average:

$$\begin{aligned}
\langle v_{\parallel,i}\rangle &= \frac{\langle (1+\delta_i(\mathbf{x}_i+\mathbf{r}_i))(1+\delta_i(\mathbf{x}_i))(u_{\parallel,i}(\mathbf{x}_i+\mathbf{r}_i)-u_{\parallel,i}(\mathbf{x}_i))\rangle}{\langle (1+\delta_i(\mathbf{x}_i+\mathbf{r}_i))(1+\delta_i(\mathbf{x}_i))\rangle} \\
&= \langle \delta_i(\mathbf{x}_i)u_{\parallel,i}(\mathbf{x}_i+\mathbf{r}_i)\rangle - \langle \delta_i(\mathbf{x}_i+\mathbf{r}_i)u_{\parallel,i}(\mathbf{x}_i)\rangle + \text{higher order terms} \\
&= -8\pi\frac{\dot{D}_i}{D_i}\int k_i j_1(k_i r_i)\mathcal{P}_i(k_i)dk_i.
\end{aligned} \tag{B3}$$

Since the power spectrum of the initial density fluctuation $\mathcal{P}_i(k_i)$ is proportional to D_i^2 (eq. [21]), the average of the initial pairwise velocity is proportional to $\dot{D}_i D_i \propto D_i^{1/2}$ and thus negligible at the limit $D_i \to 0$. If we replace D_i, $\dot{D}_i$, $\mathcal{P}_i(k_i)$ with D, $\dot{D}$, and $\mathcal{P}(k)$, respectively, the above equation gives the linear mean pairwise velocity at the time for D. This is also the case in the linear velocity dispersions obtained in §2.2 (eqs. [19] and [20]).

REFERENCES

Bond, J. R., & Efstathiou, G. 1984, ApJ, 285, L45

Bouchet, F. R., Colombi, S., Hivon, E., & Juszkiewicz, R. 1995, A&A, 296, 575

Efstathiou, G., Bond, J. R., & White, S. D. M. 1992, MNRAS, 258, 1p

Efstathiou, G., Frenk, C. S., White, S. D. M., & Davis, M. 1988, MNRAS, 235, 715

Fisher, K. B. 1995, ApJ, 448, 494

Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A., & Huchra, J. P. 1994, MNRAS, 267, 927

Górski, K. 1988, ApJ, 332, L7

Juszkiewicz, R., Fisher, K. B., & Szapudi, I. 1998, ApJ, 504, L1

Matsubara, T., Yoshisato, A., & Morikawa, M. 1998, ApJ, 504, 7

Monin, A. S., & Yaglom, A. M. 1975, Statistical Fluid Mechanics, vol. 2 (Cambridge: MIT Press)

Peacock, J. A., et al. 2001, Nature, 410, 169

Peebles, P. J. E. 1993, Principles of Physical Cosmology (Princeton: Princeton Univ. Press)

Seto, N., & Yokoyama, J. 1998, ApJ, 492, 421

Yoshisato, A., Matsubara, T., & Morikawa, M. 1998, ApJ, 498, 48

Zel'dovich, Y. B. 1970, A&A, 5, 84

Zurek, W. H., Quinn, P. J., Salmon, J. K., & Warren, M. S. 1994, ApJ, 431, 559

FIGURE CAPTIONS

Fig. 1.— Comparison of the right side of eq. (24) with the left side as a function of the separation r at $D = 1$. The solid and dashed lines denote the left and right sides of the above equation, respectively. ZA is applicable over the separations where the dashed line is above the solid line.

Fig. 2.— Two-point correlation function $\xi(r, t)$ (a) and its time derivative $\partial\xi(r, t)/\partial t$ (b) at $D = 1$ obtained with ZA. We show $\partial\xi(r, t)/\partial t$ in units of t_0^{-1}, where t_0 is the present age of the universe. The abscissa is the separation r.

Fig. 3.— Pairwise-velocity PDFs at $D = 1$. *Solid line*: Our ZA calculation at $r = 35$ Mpc. *Dashed line*: N-body simulation of Fisher et al. (1994) for $\xi(r, t) = 0.1$. The initial conditions are the same.

Fig. 4.— (a) Dependence of standard deviations of the initial relative velocities $\sigma_\parallel(r_i)$ (*solid line*) and $\sigma_\perp(r_i)$ (*dashed line*) on the separation r_i. The standard deviations are in units of $\dot{D}_i\dot{D}^{-1}$ km s^{-1}. (b) Two Gaussian functions with different standard deviations.